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Transverse momentum dependent gluon fragmentation functions from J/ψ π production at e^+e^- colliders

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Abstract The back-to-back J/ψ and π associated production at e^+e^- colliders is proposed to detect the gluon transverse momentum dependent (TMD) fragmentation functions. TMD factorization is assumed for this process. With a spinless pion, unpolarized and linearly polarized gluon TMD fragmentation functions can be defined. It is found that at parton level the hadronic tensor can be described by three independent structure functions. As a result, there are three independent angular distributions, of which the $\cos 2\phi$ azimuthal asymmetry is sensitive to the linearly polarized gluon fragmentation function.

1 Introduction

The transverse momentum dependent fragmentation function (TMDFF) is an important component of transverse momentum dependent (TMD) factorization [1,2]. It tells us how the hadron in a jet is affected by the transverse motion and polarizations of the fragmenting parton. Up to now, the quark TMDFFs have been studied very thoroughly (for a review, see [3]). But for gluon TMDFFs, the study is very limited. In this paper, we point out these functions can be extracted from quarkonium and pion associated production at e^+e^- colliders, i.e. $e^+ + e^- \to J/\psi + \pi + X$. We will use nonrelativistic QCD (NRQCD) [4] to describe the production of J/ψ. In this framework, a pair of heavy quarks is first produced from the hard interaction and then evolves into a quarkonium according to NRQCD. Thus at leading order of α_s, it must be a gluon to fragment into the final pion. If we require J/ψ and π are nearly back-to-back and there is only one jet in the final state, the relative transverse momentum distribution for J/ψ and π will be very sensitive to the transverse motion of the fragmenting gluon, and the cross section should be described by gluon TMDFFs. Since the initial state is colorless, there is no interference between initial and final states and then all soft divergences can be absorbed into the fragmentation function and NRQCD matrix elements and a proper soft factor. In this sense we expect TMD factorization to hold for this process. A potential problem is whether the fragmentation function is general, or is the same as that derived from semi-inclusive deep inelastic scattering (SIDIS). This can be checked by one-loop calculation. But here we just confine ourselves to the tree level and discuss how much information can be extracted assuming the factorization. The organization of this paper is as follows: in Sect. 2 the notations and kinematics are introduced; in Sect. 3 the formalism and the derivation of three independent angular distributions are given; Sect. 4 includes our main result and a discussion; Sect. 5 is the summary.

2 Kinematics

The process we study is

$$e^+(l') + e^-(l) \to J/\psi(P_1) + \pi(P_2) + X, \tag{1}$$

where l', l, P_1, P_2 are the momenta for each particle, respectively. For this process, it is convenient to work in the hadron frame [5]. This is a frame in the center of mass system (CMS) of the leptons and with $+z$-axis along the three momentum of the pion $\vec{P}_2$. The $+x$-axis can be an arbitrary fixed direction denoted by $\vec{n}$, which is perpendicular to $\vec{P}_2$. In this frame the momentum of lepton $\vec{l}$ is along the direction (θ, ψ), as shown in Fig. 1a. In our region of interest, J/ψ and π are nearly back-to-back, i.e. $\vec{P}^2_{1\perp} \le \Lambda^2_{\rm QCD}$. Then only the transverse direction of P_1 is relevant. The azimuthal angle between $\vec{l}_\perp$ and $\vec{P}_{1\perp}$ is defined as ϕ, as shown in Fig. 1b. Equivalently, ϕ is the angle between the plane expanded by $(\vec{P}_2, \vec{l})$ and that by $(\vec{P}_2, \vec{P}_1)$.

In our region of interest, the invariant mass of leptons $Q^2 = q^2 = (l + l')^2$ is much higher than the typical non-

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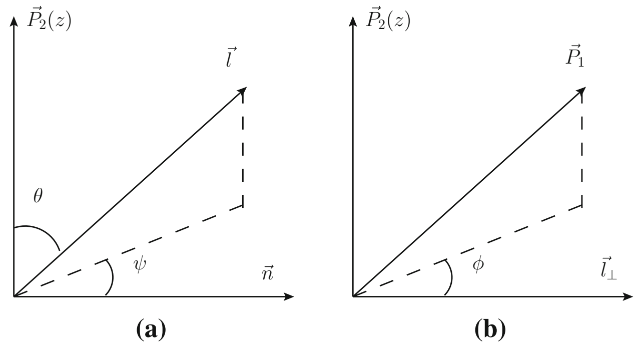

Fig. 1 The scattering angles defined in hadron frame, where the momentum of the pion P_2 is along $+z$-axis. $\vec{l}$ and $\vec{P}_1$ are the momenta of electron and J/ψ, respectively

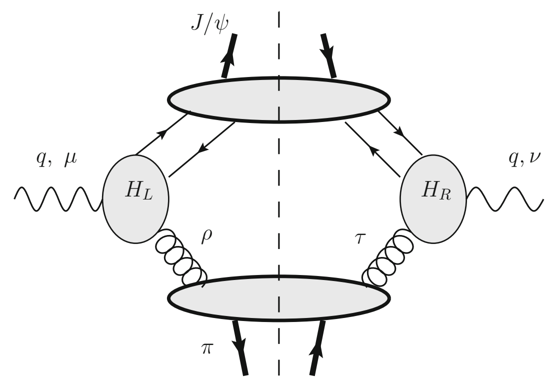

Fig. 2 The leading region for J/ψ, π back-to-back production. The *central bubbles* represent the hard scatterings

perturbative scale $\Lambda^2_{\rm QCD}$. In this region, the mass of the pion can be ignored, i.e. $P_2^2 = 0$. Since the quarkonium mass $M_J \simeq 2M_Q$ is also a hard scale, we keep the mass of J/ψ explicit in our calculation, and we define

$$\tau = \frac{M_J}{Q} = \frac{2M_Q}{Q}. \tag{2}$$

Now the TMD factorization is expected to be applicable in the region $\vec{P}^2_{1\perp} \ll Q^2$ and $\vec{P}^2_{1\perp} \ll M_J^2$.

The differential cross section can be written as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z_2 \mathrm{d}\Omega \mathrm{d}z_1 \mathrm{d}^2 P_{1\perp}} = \frac{e^2 e_Q^2 z_2}{32\pi^2 Q^4 \sqrt{z_1^2 - \tau^2}} L_{\mu\nu} W^{\mu\nu}, \tag{3}$$

where $\Omega = (\theta, \psi)$ is the solid angle of electron, e, e_Q are the electric charges of electron and heavy quark respectively, and z_1, z_2 are the energy fractions of J/ψ and π in CMS frame, that is,

$$z_1 = \frac{P_1 \cdot q}{q^2}, \quad z_2 = \frac{P_2 \cdot q}{q^2}. \tag{4}$$

The leptonic and hadronic tensors are the standard ones,

$$\begin{aligned} L_{\mu\nu} &= 2(l_\nu l'_\mu + l_\mu l'_\nu - l \cdot l' g_{\mu\nu}) - 2i\lambda \epsilon_{\mu\nu\rho\tau} l^\rho l'^\tau, \\ W^{\mu\nu} &= \sum_X \langle 0 | j^\nu(0) | J/\psi(P_1)\pi(P_2)X\rangle \\ &\quad \times \langle J/\psi(P_1)\pi(P_2)X | j^\mu(0)|0\rangle \delta^4(q - P_1 - P_2 - P_X), \end{aligned} \tag{5}$$

where $j^\mu = \bar\psi\gamma^\mu\psi$ is electromagnetic current and λ is the helicity of electron.

For the calculation, it is convenient to define the transverse direction through two light-like vectors: P_2 and $\tilde{q} \equiv q - \frac{1}{2z_2}P_2$. The transverse metric and anti-symmetric tensor are defined as

$$g_\perp^{\mu\nu} = g^{\mu\nu} - n^\mu \bar{n}^\nu - n^\nu \bar{n}^\mu, \quad \epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\tau}\bar{n}_\rho n_\tau,$$

$$n^\mu \equiv \frac{1}{\sqrt{P_2 \cdot \tilde{q}}}\tilde{q}^\mu, \quad \bar{n}^\mu \equiv \frac{1}{\sqrt{P_2 \cdot \tilde{q}}} P_2^\mu, \tag{6}$$

with $\epsilon^{0123} = 1$. In light-cone coordinates the $+$ and $-$ components of a vector a^μ are $a^+ = n \cdot a$ and $a^- = \bar{n} \cdot a$, and the transverse component is $a_\perp^\mu = g_\perp^{\mu\nu} a_\nu$.

3 Formalism and structure functions

At leading power (or twist) of $P_{1\perp}/Q$ and $P_{1\perp}/M_Q$, the cross section can be factorized into the product of a hard kernel and a fragmentation function for the pion production, as shown in Fig. 2. The fragmentation function is TMD and can be defined as [6]:

$$\begin{aligned} &\frac{1}{N_c^2 - 1}\sum_X \int \frac{\mathrm{d}\xi^- \mathrm{d}\xi_\perp^2}{(2\pi)^3} e^{ik^+\xi^- + ik_\perp \cdot \xi_\perp} \langle 0 | G^{+\tau}_{\perp a}(\xi^-, \xi_\perp) | P_h X\rangle \\ &\quad \times \langle P_h X | G^{+\rho}_{\perp a}(0)|0\rangle|_{\xi^+=0} \\ &= \frac{P_h^+}{M_h}\left[-g_\perp^{\rho\tau}\hat{G}(z, k_\perp^2) + \frac{1}{2M_h^2}(2k_\perp^\rho k_\perp^\tau - g_\perp^{\rho\tau} k_\perp \cdot k_\perp)\hat{H}(z, k_\perp^2)\right], \end{aligned} \tag{7}$$

where $G^{\mu\rho}_{\perp a}$ is the gluon field strength tensor, a is the color index. Here we have assumed the hadron h is moving along $+z$-axis, then the large component of its momentum is P_h^+. The parton momentum fraction is $1/z = k^+/P_h^+, 0 \le z \le 1$. We have suppressed the gauge links in the fragmentation functions, which are irrelevant to our discussions here. From the definition, $\hat{G}$ corresponds to the unpolarized fragmenting gluon and $\hat{H}$ corresponds to the linearly polarized gluon. Since the final hadron is spinless or unpolarized, there are only these two fragmentation functions at leading twist.

For the quarkonium production, we use NRQCD [4]. The heavy quark pair $Q\bar{Q}$ is first produced from the hard interaction, then transforms into a quarkonium. The $Q\bar{Q}$ has a small relative velocity v in the rest frame of the quark pair.

NRQCD gives a consistent power expansion according to v. To a certain power of v, only finite number of NRQCD matrix elements contribute. First, J/ψ can be written as a superposition of Fock states of the $Q\bar{Q}$ with definite angular momentum as follows [7,8]:

$$|J/\psi\rangle = \mathcal{O}(1)|Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v)|Q\bar{Q}[^3P_J^{(8)}]g\rangle \\ + \mathcal{O}(v^2)|Q\bar{Q}[^3S_1^{(1,8)}]gg\rangle \\ + \mathcal{O}(v^2)|Q\bar{Q}[^1S_0^{(8)}]g\rangle + \cdots, \qquad (8)$$

where $\cdots$ represents higher order corrections in v, and the quantum number $^{2S+1}L_J$ represents the angular momentums of the quark pair in the rest frame. For the process considered, the quark pair from hard interaction must be in color octet. From the standard power counting rule for heavy quark fields in [4], at leading order of v there are three types of NRQCD operators contributing to J/ψ production: $\mathcal{O}_8(^1S_0)$, $\mathcal{O}_8(^3S_1)$,$\mathcal{O}_8(^3P_J)$ [7,8], which correspond to the color-octet $Q\bar{Q}$ Fock states in Eq. (8). A covariant formalism is useful in the calculation. Here we use the covariant formalism in [9] to project the quark pair to definite partial waves. Suppose the momentum of heavy quark is $P = P_1/2 + q$ and that of anti-quark is $\bar{P} = P_1/2 - q$, with q a small quantity. Both quark and anti-quark are on-shell, that is, $P^2 = \bar{P}^2 = M_Q^2$. This leads to $P_1 \cdot q = 0$, and $M_J^2 = 4M_Q^2 + \mathcal{O}(q^2)$. The following operator can be used to project the amplitude to $S = 0$ wave:

$$\Pi_0 = \frac{1}{\sqrt{8M_Q^3}}(\bar{\not{P}} - M_Q)\gamma_5(\not{P} + M_Q); \qquad (9)$$

and the following operator can be used to project the amplitude to the $S = 1$ wave:

$$\Pi_1^\alpha = \frac{1}{\sqrt{8M_Q^3}}(\bar{\not{P}} - M_Q)\gamma^\alpha(\not{P} + M_Q). \qquad (10)$$

The projected partial wave amplitudes are

$$\mathcal{A}_{S=0,L=0} = \mathrm{Tr}(\mathcal{C}\Pi_0\mathcal{A}), \quad \mathcal{A}_{S=1,L=0} = \epsilon_\alpha \mathrm{Tr}(\mathcal{C}\Pi_1^\alpha\mathcal{A}), \\ \mathcal{A}_{S=0,L=1} = \epsilon_\beta \frac{\mathrm{d}}{\mathrm{d}q_\beta}\mathrm{Tr}(\mathcal{C}\Pi_0\mathcal{A})_{q=0}, \\ \mathcal{A}_{S=1,L=1} = \epsilon_{\alpha\beta}\frac{\mathrm{d}}{\mathrm{d}q_\beta}\mathrm{Tr}(\mathcal{C}\Pi_1^\alpha\mathcal{A})_{q=0}, \qquad (11)$$

where $\mathcal{C}$ is a color factor, $\mathcal{C}_{1ij} = \delta_{ij}/\sqrt{N_c}$ for color singlet and $\mathcal{C}_{8ij} = \sqrt{2}T^a_{ij}$ for color octet, a and ij are the color indices for heavy quark pair. It is clear that the amplitude $\mathcal{A}$ for heavy quark pair production does not contain the external legs for the quark pair. The total angular momentum of the quark pair can be $J = 0, 1, 2$, since here we consider all partial waves with $L \le 1$ and $S \le 1$. To obtain the cross section, we should average over the polarizations of heavy quark pair. The polarization summations are

$$\sum_{\mathrm{pol}} \epsilon_\alpha\epsilon^*_{\alpha'} = \Pi_{\alpha\alpha'} \equiv -g_{\alpha\alpha'} + \frac{P_{1\alpha}P_{1\alpha'}}{M_J^2}, \quad \epsilon^{(0)}_{\alpha\beta}\epsilon^{(0)*}_{\alpha'\beta'} = \frac{1}{3}\Pi_{\alpha\beta}\Pi_{\alpha'\beta'}, \\ \sum_{\mathrm{pol}} \epsilon^{(1)}_{\alpha\beta}\epsilon^{(1)*}_{\alpha'\beta'} = \frac{1}{2}[\Pi_{\alpha\alpha'}\Pi_{\beta\beta'} - \Pi_{\alpha\beta'}\Pi_{\alpha'\beta}], \\ \sum_{\mathrm{pol}} \epsilon^{(2)}_{\alpha\beta}\epsilon^{(2)*}_{\alpha'\beta'} = \frac{1}{2}[\Pi_{\alpha\alpha'}\Pi_{\beta\beta'} + \Pi_{\alpha\beta'}\Pi_{\alpha'\beta}] - \frac{1}{3}\Pi_{\alpha\beta}\Pi_{\alpha'\beta'}. \qquad (12)$$

These are our basic formulas to obtain partial wave amplitudes and cross sections.

According to the collinear power counting rule [10,11], at leading twist the fragmenting gluon is collinear to the final pion, i.e. $k^\mu = (k^+, k^-, k_\perp^\mu) \simeq Q(1, \lambda^2, \lambda)$ with $\lambda \simeq \Lambda_{\mathrm{QCD}}/Q \ll 1$. The leading region is shown in Fig. 2. The hadronic tensor can then be written as

$$W^{\mu\nu} = \int \mathrm{d}^4k\delta^4(q - P_1 - k)\mathcal{M}^{\mu\rho\nu\tau}_{ab} \\ \times \sum_X \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}e^{-ik\cdot\xi}\langle 0|G^b_\tau(0)|\pi X\rangle\langle\pi X|G^a_\rho(\xi)|0\rangle, \qquad (13)$$

where a, b are color indices and $\mu\nu\rho\tau$ are Lorentz indices. $\mathcal{M}^{\mu\rho\nu\tau}_{ab}$ can be expressed through the partial wave amplitudes for heavy quark pair production, i.e. $\mathcal{A}^{\mu\rho}_{a,LSJ}$, as follows:

$$\mathcal{M}^{\mu\rho\nu\tau}_{ab} = \sum_{L,S,J} \frac{1}{N_J N_{\mathrm{color}}}\mathcal{A}^{\mu\rho}_{a,LSJ}(\mathcal{A}^{\nu\tau}_{b,LSJ})^*\langle\mathcal{O}_8(^{2S+1}L_J)\rangle, \qquad (14)$$

where $\langle\mathcal{O}_8(^{2S+1}L_J)\rangle \equiv \langle 0|\mathcal{O}_8^{J/\psi}(^{2S+1}L_J)|0\rangle$ is the NRQCD matrix element for J/ψ production. Since the collinear gluon has been put into the correlation function in Eq. (13), the partial wave amplitude $\mathcal{A}^{\mu\rho}_{a,LSJ}$ does not contain the external wave functions for these collinear gluons. Here we have summed over all possible partial waves with $S \le 1$ and $L \le 1$. The average over the color and polarizations of the quark pair is necessary for production processes, since the color and polarizations have been summed over in the NRQCD matrix elements [9]. As the quark pair is in a color octet, $N_{\mathrm{color}} = N_c^2 - 1$. For different partial waves $J = 0, 1, 2$, $N_J = 1, 3, 5$ respectively.

In the collinear region, the correlation function can be written in a gauge invariant form, i.e. the gluon field just appears in the gluon field strength tensor. This can be seen as follows. First, the collinear gluon field can be decomposed into two parts:

$$G^\rho = g^{\rho\rho'} G_{\rho'} = \left(g^{\rho\rho'} - \frac{k^\rho n^{\rho'}}{k \cdot n} \right) G_{\rho'} + \frac{k^\rho n^{\rho'}}{k \cdot n} G_{\rho'}. \quad (15)$$

The contribution of the second term, after contracting with the partial wave amplitude $\mathcal{A}^{\mu\rho}_{LSJ}$, is proportional to $k_\rho \mathcal{A}^{\mu\rho}_{LSJ}$, which is just zero due to the Ward identity $\langle M|\partial^\rho G_\rho|N\rangle = 0$ where $|M\rangle$, $|N\rangle$ are on-shell quarks or physically polarized gluons. Note that here both the heavy quark and the anti-quark produced from the hard part are on-shell, even though they may have a nonzero relative momentum. So only the first term contributes. To extract the leading power contribution, we consider the following correlation function:

$$\langle G^\tau_b, G^\rho_a \rangle \equiv c \int \frac{d^4\xi}{(2\pi)^4} e^{-ik\cdot\xi} \sum_X \langle 0|G^\tau_b(0)|\pi X\rangle\langle \pi X|G^\rho_a(\xi)|0\rangle, \quad (16)$$

where the momentum k is collinear, $k^\mu = (k^+, k^-, k^\mu_\perp) \sim (1, \lambda^2, \lambda)$, and c is a certain power of M_π, which just normalizes the correlation function to be dimensionless. Since all particles are unpolarized, this correlation function just depends on k^μ. From Lorentz covariance, one can derive the following power counting rule:

$$\langle G^\tau_b, G^\rho_a \rangle \sim \frac{k^\tau k^\rho}{M^2_\pi}. \quad (17)$$

Note that $P^+_\pi = zk^+$ is the same order as k^+.

Considering the contraction with the hard part and using Eq. (15), we have

$$\langle G^\tau_b, G^\rho_a \rangle \simeq \left\langle \left(g^{\tau\tau'} - \frac{k^\tau n^{\tau'}}{k \cdot n} \right) G_{b\tau'}, \left(g^{\rho\rho'} - \frac{k^\rho n^{\rho'}}{k \cdot n} \right) G_{a\rho'} \right\rangle$$
$$= \frac{1}{(n\cdot k)^2} \langle k^\tau G^+_b - k^+ G^\tau_b, k^\rho G^+_a - k^+ G^\rho_a \rangle. \quad (18)$$

It is obvious that there is no contribution from $\tau = +$ or $\rho = +$. From the power counting rule Eq. (17), at leading power both ρ and τ should be transverse.

Then the hadronic tensor can be written as

$$W^{\mu\nu} = \int d^4k \delta^4(q - P_1 - k) \mathcal{M}^{\mu\rho\nu\tau}_{ab}$$
$$\times \frac{1}{(n\cdot k)^2} \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{-ik\cdot\xi} \langle 0|k_{\perp\tau} G^+_b(0) - k^+ G_{b\perp\tau}(0)|\pi X\rangle$$
$$\times \langle \pi X|k_{\perp\rho} G^+_a(\xi) - k^+ G_{a\perp\rho}(\xi)|0\rangle. \quad (19)$$

Now the gluon field just appears in a gluon field strength tensor, if we ignore the nonlinear term in the field strength tensor at lowest order of g_s. Since in collinear region $k^- \sim Q\lambda^2 \ll q^-$ or P^-_1, it can be ignored in the delta function and $\mathcal{M}$, and then can be integrated over in the correlation function. The resulting hadronic tensor is

$$W_{\mu\nu} = \int d^2k_\perp \delta^2(P_{1\perp} + k_\perp) \mathcal{M}^{ab}_{\mu\rho\nu\tau} \frac{1}{(n\cdot k)^2}$$
$$\times \sum_X \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-ik\cdot\xi} \langle 0|G^{+\tau}_{b\perp}(0)|\pi X\rangle\langle \pi X|G^{+\rho}_{a\perp}(\xi)|0\rangle|_{\xi^+=0}. \quad (20)$$

Since $P_{1\perp}$ is constrained to be of order $\Lambda_{\rm QCD}$, which is the same order as $k_\perp$ in collinear region, one cannot set $k_\perp$ to be zero in the delta function. Moreover, the delta function itself has already been of leading power, i.e.,

$$\int d^2k_\perp \delta^2(P_{1\perp} + k_\perp) = \mathcal{O}(1). \quad (21)$$

So one can ignore $P_{1\perp}$ and $k_\perp$ in the hard part $\mathcal{M}^{ab}_{\mu\rho\nu\tau}$ at leading power. This is a very important feature of TMD factorization.

By using the Lorentz decomposition for the correlation function Eq. (7), the hadronic tensor can be written as

$$W^{\mu\nu} = \frac{2z_2}{(1-\tau^2)Q^2 M_\pi} \delta(z_1 - z^*_1)$$
$$\times \int d^2k_\perp \delta^{(2)}(P_{1\perp} + k_\perp) \mathcal{M}^{\mu\nu}_{\rho\tau} \Bigg[- g^{\rho\tau}_\perp \hat{G}(z, k^2_\perp)$$
$$+ \frac{2k^\rho_\perp k^\tau_\perp - g^{\rho\tau}_\perp k_\perp \cdot k_\perp}{2M^2_\pi} \hat{H}(z, k^2_\perp) \Bigg] \quad (22)$$

where $\mu\nu\rho\tau$ are Lorentz indices, as shown in Fig. 2. Notice that in this formula the energy fraction of J/ψ z_1 is totally fixed at $z^*_1 = (1+\tau^2)/2$. This is a result of $P_{1\perp} \to 0$. Moreover, the momentum fraction z is determined by z_1 and z_2 definitely. With $z_1 = z^*_1$, z and z_2 satisfy a very simple relation:

$$\frac{1}{z} = \frac{1-\tau^2}{2z_2}. \quad (23)$$

As we stated before, $k_\perp$ has been set to zero in $\mathcal{M}^{\mu\nu}_{\rho\tau}$. In Eq. (22), the integration over $k_\perp$ thus can be done independently. That is,

$$\int d^2k_\perp \delta^{(2)}(P_{1\perp} + k_\perp) \Bigg[- g^{\rho\tau}_\perp \hat{G}(z, k^2_\perp)$$
$$+ \frac{2k^\rho_\perp k^\tau_\perp - g^{\rho\tau}_\perp k_\perp \cdot k_\perp}{2M^2_\pi} \hat{H}(z, k^2_\perp) \Bigg]$$
$$= -g^{\rho\tau}_\perp \mathcal{C}[w_G(k_\perp, h_\perp)\hat{G}(z, k^2_\perp)]$$
$$- (g^{\rho\tau}_\perp + 2h^\rho_\perp h^\tau_\perp) \mathcal{C}[w_H(k_\perp, h_\perp)\hat{H}(z, k^2_\perp)],$$
$$\mathcal{C}[w(k_\perp, h_\perp) f(z, k^2_\perp)]$$
$$\equiv \int d^2k_\perp \delta^{(2)}(k_\perp + P_{1\perp}) w(k_\perp, h_\perp) f(z, k^2_\perp), \quad (24)$$

where $h_\perp^\mu \equiv P_{1\perp}^\mu/\sqrt{\vec{P}_{1\perp}^2}$ satisfies $h_\perp^2 = -1$ and

$$w_G(k_\perp, h_\perp) = 1, \quad w_H(k_\perp, h_\perp) = \frac{-2(\vec{k}_\perp \cdot \vec{h}_\perp)^2 + \vec{k}_\perp^2}{2M_\pi^2}. \tag{25}$$

Substituting Eq. (24) into Eq. (22), one can see the hard part just depends on two tensors,

$$\mathcal{M}_G^{\mu\nu} = -g_\perp^{\rho\tau}\mathcal{M}_{\rho\tau}^{\mu\nu}, \quad \mathcal{M}_H^{\mu\nu} = -(g_\perp^{\rho\tau} + 2h_\perp^\rho h_\perp^\tau)\mathcal{M}_{\rho\tau}^{\mu\nu}. \tag{26}$$

From this structure, the two tensors are just two different gluon helicity amplitudes. Further simplification can be obtained by decomposing the tensor into structure functions. This can be done very easily, because the independent momenta are just q, P_2, $h_\perp$, and only $h_\perp$ is transverse. Moreover, γ_5 in the partial wave projections always appears in pair, so there will be no ϵ-tensor in $\mathcal{M}_{\rho\tau}^{\mu\nu}$ and $\mathcal{M}_{G,H}^{\mu\nu}$. To make the expressions simpler, we put $\mathcal{M}_G^{\mu\nu}$ and $\mathcal{M}_H^{\mu\nu}$ together to form a two-dimensional vector, i.e., $\mathcal{M}^{\mu\nu} = \{\mathcal{M}_G^{\mu\nu}, \mathcal{M}_H^{\mu\nu}\}$. The same rule also applies to the F_i and D_i defined later. The decomposition leads to six structure functions as follows:

$$\mathcal{M}^{\mu\nu} = F_1\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + F_2\frac{\tilde{P}_2^\mu\tilde{P}_2^\nu}{\tilde{P}_2^2} + F_3 h_\perp^\mu h_\perp^\nu + F_4\tilde{h}_\perp^\mu\tilde{h}_\perp^\nu + F_5(\tilde{P}_2^\mu q^\nu + \tilde{P}_2^\nu q^\mu) + F_6(\tilde{P}_2^\mu q^\nu - \tilde{P}_2^\nu q^\mu), \tag{27}$$

where $\tilde{P}_2 = P_2 - z_2 q$ and $\tilde{h}_\perp^\mu = \epsilon_\perp^{\mu\nu}h_{\perp\nu}$, which satisfy

$$\tilde{P}_2 \cdot q = 0, \quad g_\perp^{\mu\nu} = -h_\perp^\mu h_\perp^\nu - \tilde{h}_\perp^\mu\tilde{h}_\perp^\nu. \tag{28}$$

From Eq. (26), $h_\perp$ (or $\tilde{h}_\perp$) always appear in pair and this makes the tensor decomposition of $\mathcal{M}^{\mu\nu}$ simpler, because we need not consider the terms like $\tilde{P}_2^\mu h_\perp^\nu$. We stress that the simplification is a consequence of leading power approximation, since in $\mathcal{M}^{\mu\nu\rho\tau}$ in Eq. (22) all transverse momenta have been set to zero. The decomposition in Eq. (27) is complete but not independent, because

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} - \frac{\tilde{P}_2^\mu\tilde{P}_2^\nu}{\tilde{P}_2^2}. \tag{29}$$

Equations (28) and (29) mean the structure function F_1 is redundant and can be expressed through $F_{2,3,4}$. From QED gauge invariance $q_\mu\mathcal{M}^{\mu\nu} = q_\nu\mathcal{M}^{\mu\nu} = 0$, one has $F_5 = F_6 = 0$, which is confirmed by our calculation. Thus there are only three independent structure functions: $F_{2,3,4}$. It is clear that the F_i do not depend on the scattering angles (θ, ψ, ϕ). Thus all types of angular distribution can be obtained by contracting $\mathcal{M}^{\mu\nu}$ with leptonic tensor $L^{\mu\nu}$. The resulting cross section is

$$\frac{d\sigma}{dz_2 d\Omega dz_1 d^2P_{1\perp}} = \left(\frac{2}{3}\right)^2 \frac{2\alpha_{em}^2}{Q^4}\frac{z_2^2\delta(z_1 - z_1^*)}{(1-\tau^2)^2 M_\pi}\sum_{K=G,H}\mathcal{C}[w_K f_K] \times\left(\mathrm{D}_1^K - \frac{1}{2}\sin^2\theta\mathrm{D}_3^K + \frac{1}{2}\sin^2\theta\cos 2\phi\mathrm{D}_2^K\right), \tag{30}$$

with $f_G = \hat{G}(z, k_\perp^2)$, $f_H = \hat{H}(z, k_\perp^2)$, and $w_{G,H}$ their corresponding weights. So there are only three independent angular distributions for this process. Especially, there is a $\cos 2\phi$ azimuthal asymmetry, which will be an important signal for linear fragmentation gluon. The coefficients D_i are the superpositions of F_i as follows:

$$D_1 = F_3 + F_4 - 2F_1, \quad D_2 = F_4 - F_3, \quad D_3 = F_3 + F_4 + 2F_2, \tag{31}$$

and can be related to $\mathcal{M}_{\mu\nu}$ as follows:

$$D_1 = -g_\perp^{\mu\nu}\mathcal{M}_{\mu\nu}, \quad D_2 = (-g_\perp^{\mu\nu} - 2h_\perp^\mu h_\perp^\nu)\mathcal{M}_{\mu\nu},$$
$$D_3 = \left(-g_\perp^{\mu\nu} + 2\frac{\tilde{P}_2^\mu\tilde{P}_2^\nu}{\tilde{P}_2^2}\right)\mathcal{M}_{\mu\nu}. \tag{32}$$

NRQCD matrix elements are contained in these coefficients. To see the contribution of different partial waves, we write out the matrix elements explicitly,

$$D_i = D_i(^1S_0)\langle\mathcal{O}_8(^1S_0)\rangle + D_i(^3S_1)\langle\mathcal{O}_8(^3S_1)\rangle + \frac{1}{3}D_i(^1P_1)\langle\mathcal{O}_8(^1P_1)\rangle + \sum_{J=0}^{2} D_i(^3P_J)\frac{1}{N_J}\langle\mathcal{O}_8(^3P_J)\rangle, \quad i = 1, 2, 3. \tag{33}$$

4 Result

As we can see, the analysis in last section is independent of the details of Feynman diagrams. Since in TMD factorization the hard coefficients just receive contributions from the virtual correction [2], the formalism also applies to higher order corrections in α_s. At tree level the calculation is very simple. There are only two Feynman diagrams, as shown in Fig. 3.

After the calculation, we find that the partial waves $^3S_1$ and $^1P_1$ have no contribution to all D_i. The linearly polarized gluon fragmentation function only contributes to the $\cos 2\phi$ angular distribution, while unpolarized gluon fragmentation function does not contribute to this angular distribution at all, that is,

$$D_2^G = 0, \quad D_1^H = D_3^H = 0. \tag{34}$$

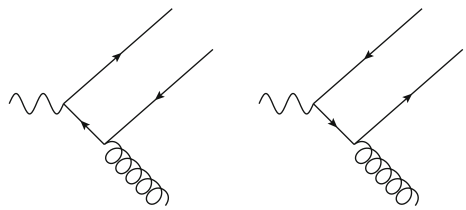

Fig. 3 The two Feynman diagrams for the hard subprocess at tree level

These two equations hold for all partial waves. Based on these facts, our final cross section can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z_2\mathrm{d}\cos\theta\mathrm{d}^2P_{1\perp}} = \left(\frac{2}{3}\right)^2\frac{4\pi\alpha_{em}^2}{Q^4}\frac{z_2^2}{(1-\tau^2)^2M_\pi}$$
$$\times\left\{\mathcal{C}[w_G\hat{G}]\left(\mathrm{D}_1^G-\frac{1}{2}\sin^2\theta\mathrm{D}_3^G\right)\right.$$
$$\left.+\mathcal{C}[w_H\hat{H}]\mathrm{D}_2^H\left(\frac{1}{2}\sin^2\theta\cos 2\phi\right)\right\}, \quad (35)$$

and

$$\left[g_s^2\frac{16(N_c^2-1)^2}{3M_Q^3(1-\tau^2)^2}\right]^{-1}\mathrm{D}_{1,3}^G = \tilde{D}_{1,3}^G(^1S_0)\langle\mathcal{O}_8(^1S_0)\rangle$$
$$+\sum_{J=0}^{2}\tilde{D}_{1,3}^G(^3P_J)\frac{1}{N_J}\langle\mathcal{O}_8(^3P_J)\rangle,$$
$$\left[g_s^2\frac{16(N_c^2-1)^2}{3M_Q^3(1-\tau^2)^2}\right]^{-1}\mathrm{D}_2^H = \tilde{D}_2^H(^1S_0)\langle\mathcal{O}_8(^1S_0)\rangle$$
$$+\sum_{J=0}^{2}\tilde{D}_2^H(^3P_J)\frac{1}{N_J}\langle\mathcal{O}_8(^3P_J)\rangle. \quad (36)$$

This is our main result. The corresponding coefficients $\tilde{D}_i$ can be found in Table 1. Notice that in Eq. (35) z_1 and ψ have been integrated over, since z_1 is just contained in a delta function and the hard coefficients are ψ independent. Now it is clear that the gluon TMD fragmentation functions can be extracted from the three independent angular distributions. By fitting the experiment data, the constraint on the involved four color-octet matrix elements can also be obtained, which are not determined very well in literature (see [12] and references therein). Two features of our tree level result should be stressed here. One is about the z_2 dependence of the hard coefficient. As can be seen from Table 1, these hard coefficients are z_2 independent. This feature will remain to higher order corrections, because the hard subprocess just depends on P_1 and q, which are z_2 independent. Another feature is the threshold enhancement which appears when $\tau^2 \to 1$. This can easily be understood. When the fragmenting gluon becomes soft, the intermediate heavy quark propagator, as shown in Fig. 3, is approaching to the mass shell, and this causes a factor $1/P_1^z$. Similar enhancement also appears in phase space integration, i.e., $\mathrm{d}P_1^z \propto Q^2\mathrm{d}z_1/P_1^z$. Since near threshold P_1^z is a small quantity proportional to $\sqrt{1-\tau^2}$, this factor results in an enhancement. This feature will also remain to higher order corrections.

Since our knowledge about gluon TMDFFs and the four involved NRQCD matrix elements is very limited, here we refrain from giving a numerical estimate of the cross section.

Before extracting these nonperturbative quantities from experiments, the most urgent task is to give a clear examination for the TMD factorization for this process at least to one-loop level, since the kinematics is so simple. At one-loop level, two types of divergence will appear. One is caused by the collinear gluon connected to the fragmenting gluon; the other is caused by the soft gluon, which may connect two heavy quarks, or one heavy quark and one fragmenting gluon, or two fragmenting gluons (real correction). For the collinear gluon, collinear power counting works, and the divergence will be absorbed into the fragmentation functions by using Ward identities since all external particles of the hard subprocess are on-shell. For the soft gluon interacting with two fragmenting gluons, the soft divergence can be absorbed into the unsubtracted TMD fragmentation functions as shown in e.g. [11]. For the soft gluon interacting with the heavy quark, the situation is a little more complicated. There are two cases. First, an octet heavy quark pair from the hard interaction may emit a real soft gluon to transmit to a color singlet quark pair. Up to the power of v we considered here, the singlet can only be $Q\bar{Q}(^3S_1)$, i.e. an S-wave singlet. In this case the soft divergence is canceled out after one sums up the divergences from heavy quark and antiquark [4]. Thus the color singlet will not affect the factorization even at one-loop level. Second, after emitting a soft gluon, the color-octet heavy quark pair may be still in a color octet. In this case, the soft divergences cannot be canceled out by summing up all relevant diagrams, since we have detected the momenta of final particles. So an additional soft factor is required. The soft factor appears

Table 1 The hard coefficients for different partial waves

	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$
$\tilde{D}_1^G(^{2S+1}L_J)$	$3Q^2\tau^2(\tau^2-1)/8$	$(1-3\tau^2)^2/2$	3	$6\tau^4+1$
$\tilde{D}_3^G(^{2S+1}L_J)$	$3Q^2\tau^2(\tau^2-1)/8$	$(1-3\tau^2)^2/2$	$-3(2\tau^2-1)$	$6\tau^4-6\tau^2+1$
$\tilde{D}_2^H(^{2S+1}L_J)$	$-3Q^2\tau^2(\tau^2-1)/8$	$(1-3\tau^2)^2/2$	-3	1

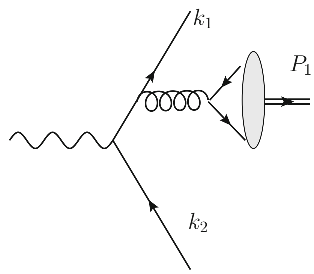

Fig. 4 One of the quark channel Feynman diagrams contributing to $J/\psi, \pi$ associated production, in which the heavy quark pair is produced by a gluon with off-shellness M_J^2 and the pion is in the anti-quark jet with momentum k_2

very naturally in TMD factorization for 2-jet (or two light hadrons) production process, i.e. $e^+e^- \to$ 2jets, because the definition of unsubtracted TMD fragmentation functions for light quark contain not only a collinear divergence, but also a soft divergence [2]. Since there is an overlap between the soft and collinear regions, the soft region contribution will be subtracted twice when we do the subtraction to obtain one-loop hard coefficients. To avoid the double subtraction, one has to introduce a soft factor in the factorization formula. For the case here, the soft divergence involves a heavy quark, so the soft factor will be very different. The form of the soft factor will be considered in detail in future work.

For the production of J/ψ and pion, one may worry that other channels also generate large contribution, such as $\gamma^* \to q\bar{q}+g^*(\to Q\bar{Q})$, where $q\bar{q}$ is a light (or heavy) quark pair, as shown in Fig. 4. The contribution is power suppressed at least by $\Lambda_{\rm QCD}^2/M_J^2$ or $P_{1\perp}^2/M_J^2$. To see this, let us assume the final state pion is produced by the decay of the anti-quark with momentum k_2. The leading power contribution still comes from the collinear region where $k_{2\perp} \sim \Lambda_{\rm QCD}$ is a small quantity. Then the conservation of a transverse momentum in the hard part leads to

$$\delta^2(k_{1\perp}+k_{2\perp}+P_{1\perp}). \tag{37}$$

The differential cross section for this diagram reads

$$\frac{d\sigma}{dz_2 d\cos\theta d^2P_{1\perp}dz_1} = \int \frac{dk_1^+}{2k_1^+}\int d^2k_{1\perp}k_1^\alpha \int d^2k_{2\perp} \\ \times \delta^2(k_{1\perp}+k_{2\perp}+P_{1\perp})\hat{q}(\tilde{z}_2,k_{2\perp})\langle 0|\mathcal{O}_8^{J/\psi}(^{2S+1}L_J)|0\rangle[\cdots]_\alpha, \tag{38}$$

where k_1^α is from the polarization summation for the final state quark, i.e., $\sum u(k_1)\bar{u}(k_1) = \not{k}_1$. $\hat{q}(\tilde{z}_2,k_{2\perp})$ is the quark TMD fragmentation function for the detected pion. $[\cdots]_\alpha$ represents the other parts of the amplitude, including the quark and gluon propagators with momenta k_1+P_1 and P_1, respectively. Since $P_1^2 = M_J^2$ is a hard scale, both propagators are off-shell at least by M_J^2. From the momentum conservation, Eq. (37), $k_{1\perp}$ can only be of order $\Lambda_{\rm QCD}$, so the volume of the phase space integration for k_1 is of order $\Lambda_{\rm QCD}^2$, i.e.,

$$\int \frac{dk_1^+}{2k_1^+}\int_{k_{1\perp}\sim\Lambda_{\rm QCD}} d^2k_{1\perp} \sim \Lambda_{\rm QCD}^2. \tag{39}$$

Since there is no enhancement from quark and gluon propagators in $[\cdots]_\alpha$, the differential cross section for this diagram is at least suppressed by $\Lambda_{\rm QCD}^2/M_J^2$. In this channel the final state light quark can also be heavy ones, but the discussion and conclusion are the same. From the argument here, one indeed needs the mass of the heavy quark to supply a power suppression and the heavy quark mass can never be ignored in the hard part, even through one needs $Q \gg M_Q$ to avoid the threshold effect. For J/ψ production, the energy of BARBA and BELLE may be suitable to extract gluon fragmentation function, $Q \simeq 10.5$ GeV; for Υ production, the suitable energy is about $Q = 15$–20 GeV. If the energy is so large that the heavy quark mass can also be ignored, the NRQCD description is not suitable, and one should use the quark fragmentation function for J/ψ (or Υ) to factorize such a differential cross section, which is just the usual TMD formalism for two-jet production.

Moreover, we can introduce an additional parameter ρ to constrain the energy of J/ψ to the endpoint region, i.e. $z_1 \in (z_1^*-\rho, z_1^*)$. The endpoint region corresponds to $\rho \to 0$. Since there is a delta function $\delta(z_1 - z_1^*)$ in the cross section of Eq. (30) for the contribution of gluon channel that we propose here, i.e., $\gamma^* \to g + Q\bar{Q}$, this channel will give a main contribution in the endpoint region. The quark channel as shown in Fig. 4 will also contribute in this region when the momentum of the final state fermion k_1^α is soft, but the contribution is obviously suppressed by k_1^α in addition to the phase space suppression. Thus we hope in this region one can obtain cleaner data to extract the gluon fragmentation functions using the following new differential cross section:

$$\sigma_\rho \equiv \int_{z_1^*-\rho}^{z_1^*} dz_1 \frac{d\sigma}{dz_2 d\cos\theta d^2P_{1\perp}dz_1}, \tag{40}$$

with $\rho \ll 1$.

Besides the process we consider here, the gluon TMD fragmentation functions may also appear in some SIDIS processes, such as $e+P \to A+B+X$ [13–15] where A, B are two hadrons almost back-to-back. It is interesting to see whether the TMD fragmentation functions there are the same as these we used here. The detailed discussion for these issues is beyond the scope of this paper and will be put into a future paper.

5 Summary

In this paper we propose the back-to-back J/ψ (or Υ) π associated production at e^+e^- colliders to extract gluon TMD fragmentation functions, making use of TMD factorization and NRQCD. The hadron frame where the final pion is along $+z$-axis is convenient for the analysis. In this frame, we find three independent angular distributions which are sensitive to the unpolarized and linearly polarized gluon TMD fragmentation functions. Especially, the linearly polarized gluon fragmentation function will contribute to the $\cos 2\phi$ azimuthal asymmetry. At tree level, NRQCD matrix elements $\langle \mathcal{O}_8(^1S_0)\rangle$ and $\langle \mathcal{O}_8(^3P_J)\rangle$ contribute to these angular distributions. Their information can also be extracted by fitting the data at e^+e^- colliders.

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